APPENDIX G

Force and Moment Calculations

G.1 Forces

The forces are calculated in CFL3D as follows. Let $\tilde{F}_l$ be the total dimensional force acting on the surface element $l$, with area $\tilde{s}_l$, normalized by $\tilde{q}_\infty \tilde{s}_{ref}$, where

$$\tilde{q}_\infty = \frac{1}{2} \tilde{\rho}_\infty |\tilde{V}|^2_\infty$$ (G-1)

and $\tilde{s}_{ref}$ is the reference area. In what follows, $\tilde{s}_{ref}$ and $\tilde{s}_l$ are taken in terms of grid dimensions. Then the dimensionless force (force coefficient) acting on element $l$ is

$$\tilde{F}_l = \frac{\tilde{F}_l}{\tilde{q}_\infty \tilde{s}_{ref}}$$ (G-2)

$\tilde{F}_l$ is composed of pressure and viscous components, $\tilde{F}_l^p$ and $\tilde{F}_l^v$, respectively. The total force coefficient is computed by summing the contributions from all specified surface elements:

$$\tilde{F} = \sum_l \tilde{F}_l$$ (G-3)

G.1.1 Pressure Component

$\tilde{F}_l^p$ is normal to the surface.

$$\tilde{s}_{ref} \tilde{F}_l^p = \frac{\tilde{p} - \tilde{p}_\infty}{\frac{1}{2} \tilde{\rho}_\infty |\tilde{V}|^2_\infty} \tilde{s}_l \tilde{n} = 2 \frac{\tilde{p}/(\tilde{\rho}_\infty \tilde{a}_\infty^2) - \tilde{p}_\infty/(\tilde{\rho}_\infty \tilde{a}_\infty^2)}{|\tilde{V}|^2_\infty / \tilde{a}_\infty^2} \tilde{s}_l \tilde{n} = 2 \frac{p - 1}{M_\infty^2} \gamma \tilde{s}_l \tilde{n}$$ (G-4)

Therefore
\[ \tilde{s}_{\text{ref}} \tilde{F}_l^p = \frac{2}{\gamma M_\infty^2} (\gamma p - 1) \tilde{s}_l \hat{n} \]  

(G-5)

The \( x \), \( y \), and \( z \) components of \( \tilde{F}_l^p \) are obtained by multiplying Equation (G-5) by the appropriate direction cosine. For example, if element \( l \) lies on an \( i = \) constant surface,

\[ \hat{n} = \nabla \hat{\xi} \]

(G-6)

\[ \tilde{s}_{\text{ref}} \tilde{F}_l^p \bigg|_x = \frac{2}{\gamma M_\infty^2} (\gamma p - 1) \tilde{s}_l \hat{\xi}_x \]

\[ \tilde{s}_{\text{ref}} \tilde{F}_l^p \bigg|_y = \frac{2}{\gamma M_\infty^2} (\gamma p - 1) \tilde{s}_l \hat{\xi}_y \]

\[ \tilde{s}_{\text{ref}} \tilde{F}_l^p \bigg|_z = \frac{2}{\gamma M_\infty^2} (\gamma p - 1) \tilde{s}_l \hat{\xi}_z \]

(G-7)

where \( \hat{\xi}_x = \xi_x / |\nabla \xi| \), \( \hat{\xi}_y = \xi_y / |\nabla \xi| \), and \( \hat{\xi}_z = \xi_z / |\nabla \xi| \).

G.1.2 Viscous Component

\( \tilde{F}_l^v \) is tangential to the surface.

\[ \tilde{s}_{\text{ref}} \tilde{F}_l^v = \frac{\tilde{\xi}}{\tilde{V}_\infty \sqrt{\tilde{a}_\infty^2}} \tilde{s}_l = \frac{\tilde{\xi}/(\tilde{\rho}_\infty \tilde{a}_\infty^2)}{\sqrt{\tilde{V}_\infty^2 / \tilde{a}_\infty^2}} \tilde{s}_l = \frac{\tilde{s}_l}{M_\infty^2} \]

(G-8)

Consider the flow near a surface element \( l \) of an \( i = \) constant surface. (See Figure G-1.)

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**Figure G-1.** Viscous force component example.
In the figure,

\[ \tilde{V} = \text{velocity vector} \]
\[ \tilde{V}_n = \text{normal velocity vector} \]
\[ \tilde{V}_t = \text{tangential velocity vector (0 on the surface)} \]

and

\[ \tilde{V}_t = \tilde{V} - \tilde{V}_n \] (G-10)

\[ \tilde{\tau}_t = \tilde{\mu} \frac{\partial \tilde{V}_t}{\partial \tilde{n}} = \tilde{\mu} \frac{\partial (\tilde{V} - \tilde{V}_n)}{\partial \tilde{n}} \] (G-11)

where

\[ \tilde{V}_n = (\tilde{V} \cdot \hat{n}) \hat{n} \] (G-12)

(\( \tilde{V} \cdot \hat{n} \) is the normalized contravariant velocity and \( \hat{n} \) is the unit surface normal.) The \( x \) component of \( \tilde{\tau}_t \) is \( \tilde{\tau}_{tx} \):

\[ (\tilde{\tau}_{tx}) = \tilde{\mu} \frac{\partial \tilde{V}_t}{\partial \tilde{n}} \tilde{\xi}_x \] (G-13)

Similarly,

\[ (\tilde{\tau}_{ty}) = \tilde{\mu} \frac{\partial \tilde{V}_t}{\partial \tilde{n}} \tilde{\xi}_y \] (G-14)

\[ (\tilde{\tau}_{tz}) = \tilde{\mu} \frac{\partial \tilde{V}_t}{\partial \tilde{n}} \tilde{\xi}_z \] (G-15)

Consider the nondimensionalization of the \( x \) component of \( \tilde{\tau}_t \):
\[
(\tilde{\tau}_l)_x = \frac{(\tilde{\tau}_l)_x}{\tilde{\rho}_\infty \bar{\alpha}_\infty^2} = \frac{1}{\tilde{\rho}_\infty \bar{\alpha}_\infty^2} \cdot \bar{\mu} \left[ \tilde{u} - \left( \tilde{\nabla} \cdot \tilde{n} \right) \tilde{\xi}_x \right]
\]  

(See Equation (G-8).) Then

\[
(\tilde{\tau}_l)_x = \frac{\bar{\mu}_\infty |\tilde{V}|_\infty \tilde{\mu} \left\{ \left[ \tilde{u} - \left( \tilde{\nabla} \cdot \tilde{n} \right) \tilde{\xi}_x \right] / \tilde{\alpha}_\infty \right\}}{\tilde{\rho}_\infty |\tilde{V}|_\infty \bar{L}_R \bar{\alpha}_\infty \bar{\mu}_\infty \partial (\tilde{n} / \bar{L}_R)}  
\]

Therefore,

\[
(\tilde{\tau}_l)_x = \frac{M_\infty \bar{\mu} \partial [u - (\tilde{V} \cdot \tilde{n}) \tilde{\xi}_x]}{Re \bar{L}_R}
\]

with similar expressions for \((\tilde{\tau}_l)_y\) and \((\tilde{\tau}_l)_z\). The derivative is evaluated using the cell-center and wall values of a cell volume like that shown in Figure G-2.

![Figure G-2. Cell volume example.](image)

That is, for the \(x\) component,

\[
\frac{\partial \left[ \tilde{u} - \left( \tilde{\nabla} \cdot \tilde{n} \right) \tilde{\xi}_x \right]}{\partial n} = \frac{\left[ \tilde{u} - \left( \tilde{\nabla} \cdot \tilde{n} \right) \tilde{\xi}_x \right]_c - 0}{\frac{1}{2} \Delta n}
\]

where the subscript \(c\) denotes the cell-center value and \(\left[ \tilde{u} - \left( \tilde{\nabla} \cdot \tilde{n} \right) \tilde{\xi}_x \right]_c \equiv 0\) on a solid wall with the no-slip assumption. So
Thus, for the $x$ component of the viscous force,

\begin{equation}
(\ddot{\tau}_l)_x = \frac{M_\infty}{Re_{LR}} \mu \left[ \tilde{u} - (\tilde{V} \cdot \hat{n}) \tilde{\xi}_x \right]_c = \frac{2M_\infty}{Re_{LR}} \mu \left[ \tilde{u} - (\tilde{V} \cdot \hat{n}) \tilde{\xi}_x \right]_c \tag{G-20}
\end{equation}

Similarly,

\begin{equation}
\tilde{s}_{\text{ref}}(\ddot{F}_l)_x = \frac{2}{M_\infty^2} (\ddot{\tau}_l)_x \tilde{s}_l = \frac{4}{M_\infty Re_{LR}} \mu [u - (\tilde{V} \cdot \hat{n}) \tilde{\xi}_x]_c \frac{\tilde{s}_l^2}{\tilde{v}_l} \tag{G-21}
\end{equation}

Similarly,

\begin{equation}
\tilde{s}_{\text{ref}}(\ddot{F}_l)_y = \frac{4}{M_\infty Re_{LR}} \mu [v - (\tilde{V} \cdot \hat{n}) \tilde{\xi}_y]_c \frac{\tilde{s}_l^2}{\tilde{v}_l} \tag{G-22}
\end{equation}

\begin{equation}
\tilde{s}_{\text{ref}}(\ddot{F}_l)_z = \frac{4}{M_\infty Re_{LR}} \mu [w - (\tilde{V} \cdot \hat{n}) \tilde{\xi}_z]_c \frac{\tilde{s}_l^2}{\tilde{v}_l} \tag{G-23}
\end{equation}

**G.2 Moments**

The moments due to the forces acting on an element are determined as follows. Let

\begin{align*}
(\ddot{F}_l)_x &= (\ddot{F}_l^p + \ddot{F}_l^v)_x \\
(\ddot{F}_l)_y &= (\ddot{F}_l^p + \ddot{F}_l^v)_y \\
(\ddot{F}_l)_z &= (\ddot{F}_l^p + \ddot{F}_l^v)_z \tag{G-24}
\end{align*}

Figure G-2 illustrates the directions of the moments. All moments are positive if counter-clockwise when viewed from the positive axis. The conventions (assuming $x$ points downstream and $z$ points up) are

$M_x$: rolling moment; positive for counter-clockwise roll when viewed from downstream.

$M_y$: pitching moment; positive for pitch up.

$M_z$: yawing moment; positive for counter-clockwise yaw when viewed from above. Therefore,
Note that the reference length used to nondimensionalize \((\vec{M}_l)_y\) is \(\tilde{b}_{ref}\), while \((\vec{M}_l)_z\) is made dimensionless with \(\tilde{b}_{ref}\). This is the default for CFL3D-type grids or PLOT3D-type grids with \texttt{ialph} = 0 (see “LT3 - Flow Conditions” on page 19). However, if a PLOT3D-type grid is used with \texttt{ialph} = 1, then \((\vec{M}_l)_y\) is made nondimensional with \(\tilde{b}_{ref}\) and \((\vec{M}_l)_z\) is made dimensionless with \(\tilde{c}_{ref}\). \((\vec{M}_l)_x\) is always made dimensionless with \(\tilde{b}_{ref}\). By switching the reference lengths based on \texttt{ialph}, the moment coefficient that is normally associated with the pitching moment is always based on \(\tilde{c}_{ref}\), while the moment coefficient that is normally associated with the yawing moment is always based on \(\tilde{b}_{ref}\). Because \((\vec{M}_l)_x\) always uses \(\tilde{b}_{ref}\), the moment coefficient associated with the rolling moment is based on \(\tilde{b}_{ref}\).